Radiative energy loss reduction in an absorptive plasma

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Abstract. The influence of the damping of radiation on the radiative energy loss spectrum of a relativistic charge in an infinite, absorptive plasma is studied. We find increasing reduction of the spectrum with increasing damping. Our studies, which represent an Abelian approximation for the colour charge dynamics in the quark-gluon plasma, may influence the analysis of jet quenching phenomena observed in high-energy nuclear collisions. Here, we focus on a formal discussion of the limiting behaviour with increasing radiation frequency. In an absorptive (and polarizable) medium, this is determined by the behaviour of the exponential damping factor entering the spectrum and the formation time of radiation.

1. Introduction

The strong quenching of jets observed experimentally in high-energy nuclear collisions at the Relativistic Heavy Ion Collider [1, 2, 3, 4] and the Large Hadron Collider [5] has been interpreted as an indication for the formation of an opaque, deconfined plasma of QCD matter, in which the leading partons suffer medium-induced energy loss. The energy loss of relativistic partons is expected to be dominated by gluon radiation [6, 7, 8]. Important in this context is the possibility of a destructive interference between the radiation amplitudes of multiple deflections during the formation time of gluons [9] and the influence of the dielectric polarization of the plasma [10, 11]. Both effects lead to a modification of the gluon radiation spectrum and, consequently, of the radiative energy loss. So far, however, the possible influence of damping of radiation in an absorptive plasma has been neglected in these considerations.

2. Radiative energy loss spectrum

Recently [12], the impact of polarization and absorption effects in a medium on the radiative energy loss of a relativistic charge q has been studied in linear response theory. Thereby, the influence of multiple deflections of the charge is incorporated in line with the original approach in [13]. These investigations can be viewed as abelian approximation for the dynamics of a colour charge in the QCD plasma. Nonetheless,

they miss still the important non-abelian contributions from gluon rescatterings [9]. The absorptive, dielectric medium is phenomenologically modelled by a complex index of refraction squared of the form $n^2(\omega) = 1 - m^2/\omega^2 + 2i\Gamma/\omega = [n_r(\omega) + in_i(\omega)]^2$ with frequency ω, and m and Γ accounting for a finite in-medium gluon mass [10, 11] and damping rate [14, 15], respectively. For an infinite medium with permeability $\mu(\omega) = 1$, and in the case n_r and n_i have equal signs, the radiative energy loss spectrum per unit length reads [12]

$$-\frac{d^2W}{dzd\omega} \simeq -\frac{2\alpha}{3\pi}\frac{\hat{q}}{E^2}\int_0^\infty d\bar{t}\,\omega\cos(\omega\bar{t}\,)\sin\left[\omega|n_r|\beta\bar{t}\left(1-\frac{\hat{q}\bar{t}}{6E^2}\right)\right]\mathcal{F}(\bar{t}\,)\,. \quad (1)$$

Here, with $\hbar = c = 1$, $\alpha = q^2/(4\pi)$ is the coupling, $\beta = \sqrt{1-1/\gamma^2}$ with $\gamma = E/M$ for a charge with energy E and mass M, $\mathcal{F}(t) = \exp[-\omega|n_i|\beta t(1 - \hat{q}t/(6E^2))]$ is the exponential damping factor related to the imaginary part of $n(\omega)$ and the parameter $\hat{q}$ denotes the mean accumulated transverse momentum squared of the deflected charge per unit time.

In the vacuum limit, i.e., when setting $n_r = 1$ and $n_i = 0$, the dominant contribution of Eq. (1) for β close to 1 reads

$$-\frac{d^2W}{dzd\omega} \simeq \frac{2\alpha}{3\pi}\frac{\hat{q}}{M^2}\int_0^\infty dx\sin\left[x+\frac{2\hat{q}x^2}{3\omega M^2(1-\beta^2)}\right]. \quad (2)$$

This coincides with the result for the radiation intensity derived in [13], when $\hat{q}$ is identified with the parameters used in [13]. With increasing ω, Eq. (2) approaches, for $\hat{q}E/M^4 \ll 1$, the ω-independent limit

$$-\frac{d^2W}{dzd\omega} \simeq \frac{2\alpha\hat{q}}{3\pi M^2}\,. \quad (3)$$

The limiting behaviour Eq. (3) as well as the full vacuum spectrum Eq. (2) are shown for specific kinematic parameters in Fig. 1 (left panel) by dash-dotted and solid curves, respectively. As evident, the limit Eq. (3) is approached rather quickly by the full vacuum spectrum.

In Fig. 1 (left panel), also the full spectrum Eq. (1) for an absorptive medium with $m = 0$ and different, finite values of Γ is exhibited by dashed curves. Even for small Γ, the spectrum is significantly reduced compared to the vacuum result in the region of small and intermediate ω. This suppression of the spectrum is a consequence of the exponential damping factor in Eq. (1), cf. the discussion in [12]. In the limit $\omega \gg \Gamma$ and $m = 0$, the dominant contribution to Eq. (1) for β close to 1 is given by

$$-\frac{d^2W}{dzd\omega} \simeq -\frac{\alpha}{3\pi}\frac{\hat{q}}{E^2}\int_0^\infty d\bar{t}\,\omega\sin\left[\omega\bar{t}(\beta-1)-\frac{\omega\beta\hat{q}\bar{t}^2}{6E^2}\right]\mathcal{F}(\bar{t}\,)\,. \quad (4)$$

It is interesting to study whether this damped spectrum Eq. (4) formally approaches the same limit Eq. (3) for increasing ω as the vacuum result. For this to happen, $\mathcal{F}(t)$ would have to become 1 with increasing ω within the time interval, which is essential for the integral in Eq. (4). In the considered limit $\omega \gg \Gamma$, $\omega|n_i| \to \Gamma$, such that $\mathcal{F}(t) \to \exp[-\Gamma\beta t(1 - \hat{q}t/(6E^2))]$, which is *per se* ω-independent as long as Γ does not

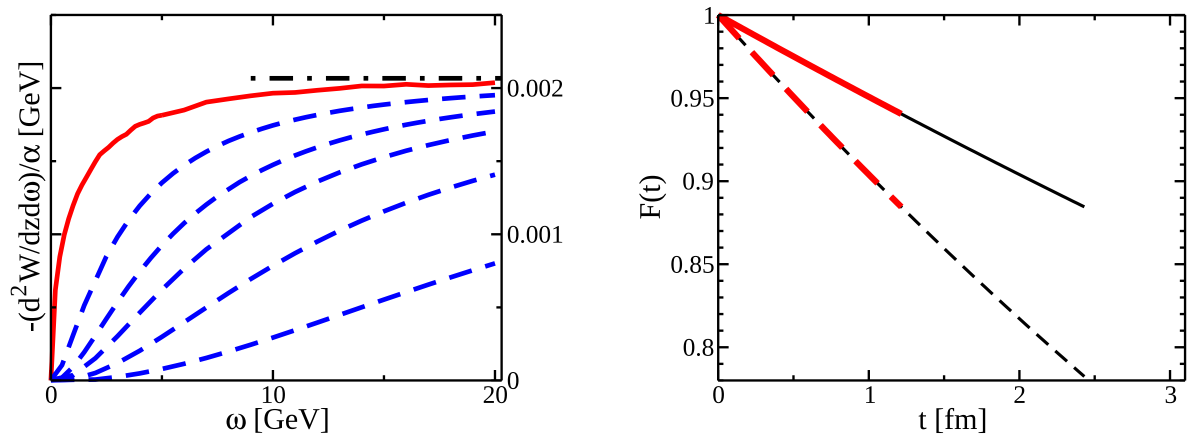

Figure 1. Left: radiative energy loss spectrum per unit length as a function of ω for a charge with $E = 50$ GeV, $M = 4.5$ GeV and $\hat{q} = 1$ GeV$^2/fm$. The solid curve depicts the vacuum result Eq. (2) as a reference, while the dash-dotted curve shows its formal limit for increasing ω, Eq. (3). The dashed curves exhibit the full, reduced spectrum Eq. (1) in an absorptive medium with $m = 0$ and $\Gamma = 10, 20, 30, 50, 100$ MeV from top to bottom. Right: exponential damping factor $\mathcal{F}(t)$ as a function of t within the essential time interval of the integral in Eq. (4) determined by t_f. The kinematic parameters E, M and $\hat{q}$ are chosen as in the left panel. Upper solid curves are for $\Gamma = 10$ MeV and lower dashed curves for $\Gamma = 20$ MeV, while the thinner curves are for $\omega = 20$ GeV and the thicker curves on top of them are for $\omega = 40$ GeV.

depend on ω and, thus, would imply a modification of the spectrum for any ω. However, the essential time interval itself depends on the frequency, such that the influence of $\mathcal{F}(t)$ on the spectrum Eq. (4) varies with ω. The essential time interval may be determined by the formation time t_f of radiation, which for not too large Γ and/or γ, cf. the discussion in [12, 16], can be found from Eq. (4) by a condition for the phase factor

$$\Phi(t_f) \equiv \omega t_f(1-\beta) + \frac{\omega\beta\hat{q}}{6E^2}t_f^2 \sim 1\,. \qquad (5)$$

A rough estimate for the solution t_f of this condition equation is given by the minimum of $t_f^{(s)} \sim 1/[\omega(1-\beta)]$ and $t_f^{(m)} \sim E\sqrt{6/(\omega\beta\hat{q})}$. For γ small compared to $M^3/\hat{q}$, this minimum is, with increasing ω, given by $t_f \sim t_f^{(s)}$. Then, with $\mathcal{F}(t) \sim \exp[-\Gamma t]$ for relativistic particles, the exponential damping factor decreases monotonically from 1 at $t = 0$ to $\exp[-\Gamma/(\omega(1-\beta))]$ at $t \sim t_f$. The behaviour of $\mathcal{F}(t)$ as a function of t is exhibited in Fig. 1 (right panel) for different ω and different Γ. As evident, increasing Γ will increase the influence of $\mathcal{F}(t)$ on Eq. (4). With increasing ω, t_f decreases and, consequently, the decrease of $\mathcal{F}(t)$ plays less and less role in Eq. (4). For $\omega \to E$, $\mathcal{F}(t_f) \to \exp[-\Gamma E/M^2]$, which is in general different from 1. This implies that the asymptotic limit Eq. (3) can be recovered for large ω only provided $E \ll \min\,(M^4/\hat{q}, M^2/\Gamma)$.

For $\gamma \gtrsim M^3/\hat{q}$, $t_f \sim t_f^{(m)}$ even for $\omega \to E$, such that $\mathcal{F}(t_f) \sim \exp[-\Gamma E/\sqrt{\omega\hat{q}}] \to \exp[-\Gamma\sqrt{E/\hat{q}}]$ for $\omega \to E$, implying qualitatively the same behaviour of $\mathcal{F}(t)$ as

discussed before. Moreover, this illustrates the increasing importance of the exponential damping factor on the spectrum with increasing E, cf. [12].

Similar observations can be made in the case of a medium with $m \neq 0$ in the limit $\omega \gg m \gg \Gamma$. Then, $n_r(\omega) \rightarrow 1$ rapidly with increasing ω such that Eq. (1) reduces likewise to Eq. (4) for $\beta \rightarrow 1$. Details in the limiting behaviour depend, though, on details in the formation time, which is influenced by finite m-effects, cf. [16].

3. Conclusions, and Acknowledgments

In summary, we have presented results for the radiative energy loss spectrum of a relativistic charge undergoing multiple scatterings in an absorptive medium. Our investigations may be viewed as abelian approximation for the dynamics of a colour charge in an infinite deconfined QCD plasma. But they are applicable also, if the system size is large compared to the formation length of radiation. These studies, however, still miss the important non-abelian contributions from gluon rescatterings. We find an increasing reduction of the spectrum compared to the vacuum case for increasing damping of radiation in a purely absorptive plasma. In this paper, special emphasis is put on the discussion of the limiting behaviour of the spectrum with increasing ω. For an absorptive medium, this is driven by the behaviour of an exponential damping factor in the spectrum, which is related to the formation time of radiation in the matter.

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